

Simulation of Motion of Satellite under the Effect of Oblateness of Earth and Atmospheric Drag

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ABSTRACT

The equations governing motion of the satellite under the effect of oblateness of Earth and atmospheric drag have been simulated, for a fixed initial position and three different initial velocities, till satellite collapses on Earth. Simulation of motion of artificial Earth satellite subject to the combined effects of oblate Earth and atmospheric drag is presented. The atmospheric model considered here takes in to account of exponential variation of the density with initial distance of Satellite from Earth's surface, scale height and radial distance. The minimum and maximum values of orbital elements and their variation over a time for different initial velocities have been reported.

Subject headings: Artificial satellite; Oblateness of Earth; Atmospheric drag; Motion of satellite

1. Introduction

The study of motion of the satellite and its life span is the topic of interest of many researchers over the past few decades. When the orbit of the satellite is low Earth orbit (LEO), the perturbation due to oblateness of Earth and atmospheric drag plays very important role. Various analytic, semi-analytic and numerical techniques are adopted for solving perturbed equations of motion. Sharma and Raj (2007) extensively studied the motion of satellite under the oblateness of Earth and also by considering atmospheric drag. They solved the equations of motion by applying KS transformations (Stiefel and Scheifele (1972)). King-Hele (1958) solved the equations of the motion of a satellite analytically by considering oblateness of Earth. The motion of satellite in the terrestrial upper atmosphere was studied by Sehnal (1980). Knowles *et. al.* (2001) analyzed the effect of geomagnetic storm's driven by solar eruption on upper atmosphere of Earth and its effect on motion of satellite. The dynamics of satellite motion around the oblate Earth using rotating frame were developed by Yan and Kapila (2001). The Hamilton equations for the motion of satellite under the Earth's oblateness and atmospheric drag were derived and solved using canonical transformation by Khalil (2002). Bezdvěk and Vokrouhlický (2004) presented a semi-analytic theory for long-term dynamics of a low Earth orbit of artificial satellites, they considered both oblateness of Earth and atmospheric drag. Some statistical measures were used by them to compare the observations over the computer efficiency. The resonance in satellite motion under air drag was studied by Bhardwaj and Sethi (2006). Hassan *et. al.* (2008) tried to find a solution of equations governing motion of artificial satellites under the effect of an oblateness of Earth by using KS variables. The authors then applied Picard's iterative method to find the solution. The algorithm is prescribed by the authors depends on initial guess solution. The differential equations governing relative motion of the satellite under the oblateness of Earth and atmospheric drag were derived and solved by Chen and Jing (2010), the wide application of their work is in satellite attitude

control and orbital maneuver for inter-planetary missions. The satellite rotational dynamics was studied and simulated by Lee *et. al.* (2010) using Lie group variational integrator approach. Reid and Misra (2011) studied the effect of aerodynamic forces on the formation flight of satellite. The analytic solution in terms of Keplerian angular elements of satellite orbit under atmospheric drag was studied by Xu *et. al.* (2011). Al-Bermani *et. al.* (2012) investigated the effect of atmospheric drag and zonal harmonic J_2 for the near Earth orbit satellite namely *Cosmos1484*. The analytic solution of motion of satellite by considering combined effect of Earth's gravity and air drag was found by Delhaise (1991) using Lie transformations. Aghav and Gangal (2014) designed and simplified the orbit determination algorithm for low Earth orbit navigation.

In this paper we have simulated the equations of motion of satellite under the influence of oblateness of Earth and atmospheric drag in the low Earth orbit. We have analyzed results for 1–day, 1–month, 6–months and till satellite collapses on Earth, by considering fixed initial position and three different initial velocities. The orbital elements have been computed for the above mentioned period. We have considered the initial velocities in such a way that satellite will remain in low Earth orbit.

The paper is organized as follows: section 2 describes the model. Solution of equations governing motion of the satellite under oblateness of Earth and atmospheric drag and calculation of orbital elements are reported in section 3. Section 4 contains discussion and concluding remarks.

2. The Model

The equations of motion of satellite without any additional perturbing force other than gravitational force between Earth and satellite is given by

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}, \quad (1)$$

where $\mu = GM$, G is gravitational constant and M is mass of Earth. In the presence of perturbation, we need to add perturbing acceleration on the right side of equation (1).

Since we are considering perturbation due to oblateness of Earth and perturbation due to atmospheric drag, the equations of motion can be written as

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_O + \vec{a}_A, \quad (2)$$

where \vec{a}_O is acceleration due to oblateness of Earth and \vec{a}_A is acceleration due to atmospheric drag. The second order equation (2) can be written as following set of two first order differential equations

$$\begin{aligned} \dot{\vec{r}} &= \vec{v}, \\ \dot{\vec{v}} &= -\frac{\mu}{r^3}\vec{r} + \vec{a}_O + \vec{a}_A. \end{aligned} \quad (3)$$

In the Cartesian co-ordinate system the system of equations (3) takes the form,

$$\begin{aligned} \dot{x} &= v_x, \\ \dot{y} &= v_y, \\ \dot{z} &= v_z, \\ \dot{v}_x &= -\frac{\mu x}{r^3} + \vec{a}_{O_x} + \vec{a}_{A_x}, \\ \dot{v}_y &= -\frac{\mu y}{r^3} + \vec{a}_{O_y} + \vec{a}_{A_y}, \\ \dot{v}_z &= -\frac{\mu z}{r^3} + \vec{a}_{O_z} + \vec{a}_{A_z}, \end{aligned} \quad (4)$$

where $\vec{a}_{O_x}, \vec{a}_{O_y}$ and \vec{a}_{O_z} are components of acceleration due to oblateness of Earth in the direction x, y and z axis respectively and $\vec{a}_{A_x}, \vec{a}_{A_y}$ and \vec{a}_{A_z} are components of acceleration

due to atmospheric drag in x, y and z axis respectively.

The Earth's gravitational potential can be modeled in terms of zonal harmonics (Battin (1987)). In the expression the value of J_2 zonal coefficient is 400 times higher than other J_n zonal coefficient, $n \geq 3$. Hence we consider only J_2 into account. If these higher order zonal coefficients are neglected and taking the gradient of scalar potential function then the components of acceleration due to oblateness of Earth in the direction of x, y and z direction respectively are,

$$\begin{aligned}\vec{a}_{O_x} &= -\frac{3\mu R^2 J_2 x (x^2 + y^2 - 4z^2)}{2r^7}, \\ \vec{a}_{O_y} &= -\frac{3\mu R^2 J_2 y (x^2 + y^2 - 4z^2)}{2r^7}, \\ \vec{a}_{O_z} &= -\frac{3\mu R^2 J_2 z (3x^2 + 3y^2 - 2z^2)}{2r^7},\end{aligned}\tag{5}$$

where $R = 6378.1363$ is radius of Earth, $\mu = GM = 398600.436233 \text{ km}^3/\text{sec}^2$ and $J_2 = 1082.63 \times 10^{-6}$.

The acceleration due to atmospheric density is given by

$$\vec{a}_A = -\frac{1}{2}\rho \frac{C_D A}{m} |\vec{v}_r| \vec{v}_r,\tag{6}$$

where ρ is atmospheric density, C_D is drag coefficient, A is cross sectional area of the satellite perpendicular to velocity vector, m is mass of satellite and \vec{v}_r is satellite velocity vector relative to an atmosphere.

We take the simple exponential atmospheric model for which atmospheric density given by,

$$\rho = \rho_{pa} e^{\left[\frac{(r_{pa} - r)}{H}\right]},\tag{7}$$

where ρ_{pa} is the density at initial perigee point, r_{pa} is the initial distance of satellite from Earth's surface, $r = |\vec{r}|$ and H is scale height. The ratio $B^* = \frac{m}{C_D A}$ is called the Ballistic coefficient.

We assume that the atmosphere rotates at the same angular speed as Earth. With this

assumption the relative velocity vector is given by Wiesel (2003)

$$\vec{v}_r = \vec{v} - \vec{\omega} \times \vec{r}, \quad (8)$$

where, $\vec{\omega}$ is the inertial rotation vector of the Earth given by

$$\vec{\omega} = \omega_e \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (9)$$

where, $\omega_e = 7.292115486 \times 10^{-5} \text{ rad/sec}$. The cross product of the (8) and (9) gives three components of the relative velocity vector as

$$\vec{v}_r = \begin{bmatrix} v_x + \omega_e r_y \\ v_y - \omega_e r_x \\ v_z \end{bmatrix}. \quad (10)$$

Substituting (7), (10) and B^* in (6), we get the components of acceleration due to atmospheric drag in the direction of x, y and z axis respectively as

$$\begin{aligned} a_{A_x} &= -\frac{\rho_{pa} e^{\left[\frac{r_{pa}-r}{H}\right]} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2} (v_x + \omega_e r_y)}{2B^*}, \\ a_{A_y} &= -\frac{\rho_{pa} e^{\left[\frac{r_{pa}-r}{H}\right]} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2} (v_y - \omega_e r_x)}{2B^*}, \\ a_{A_z} &= -\frac{\rho_{pa} e^{\left[\frac{r_{pa}-r}{H}\right]} \sqrt{(v_x + \omega_e r_y)^2 + (v_y - \omega_e r_x)^2 + v_z^2} v_z}{2B^*}. \end{aligned} \quad (11)$$

Substituting (5) and (11) in (4), we get equations of motion of satellite under oblateness of Earth and atmospheric drag as

$$\begin{aligned}
 \dot{x} &= v_x, \\
 \dot{y} &= v_y, \\
 \dot{z} &= v_z, \\
 \dot{v}_x &= -\frac{\mu x}{r^3} - \frac{3\mu R^2 J_2 x(x^2 + y^2 - 4z^2)}{2r^7} - \frac{\rho_{pa} e^{[\frac{r_{pa}-r}{H}]} \sqrt{(v_x + \omega_e r_y)^2 + (vy - \omega_e r_x)^2 + v_z^2} (v_x + \omega_e r_y)}{2B^*}, \\
 \dot{v}_y &= -\frac{\mu y}{r^3} - \frac{3\mu R^2 J_2 y(x^2 + y^2 - 4z^2)}{2r^7} - \frac{\rho_{pa} e^{[\frac{r_{pa}-r}{H}]} \sqrt{(v_x + \omega_e r_y)^2 + (vy - \omega_e r_x)^2 + v_z^2} (vy - \omega_e r_x)}{2B^*}, \\
 \dot{v}_z &= -\frac{\mu z}{r^3} - \frac{3\mu R^2 J_2 z(3x^2 + 3y^2 - 2z^2)}{2r^7} - \frac{\rho_{pa} e^{[\frac{r_{pa}-r}{H}]} \sqrt{(v_x + \omega_e r_y)^2 + (vy - \omega_e r_x)^2 + v_z^2} v_z}{2B^*}.
 \end{aligned} \tag{12}$$

For the exponential atmospheric model the scale height (H) and ρ_{pa} can be computed from the table-1 (Vallado (2004)).

Table 1: Density at Initial Perigee Point and Scale Height

r (km)	r_{pa}	ρ_{pa} (kg/m ³)	H (km)
0 – 25	0	1.225	7.249
25 – 30	25	3.899×10^{-2}	6.349
30 – 40	30	1.774×10^{-2}	6.682
40 – 50	40	3.972×10^{-3}	7.554
50 – 60	50	1.057×10^{-3}	8.382
60 – 70	60	3.206×10^{-4}	7.714
70 – 80	70	8.770×10^{-5}	6.549
80 – 90	80	1.905×10^{-5}	5.799
90 – 100	90	3.396×10^{-6}	5.382
100 – 110	100	5.297×10^{-7}	5.877
110 – 120	110	9.661×10^{-8}	7.263
120 – 130	120	2.438×10^{-8}	9.473
130 – 140	130	8.484×10^{-9}	12.636
140 – 150	140	3.845×10^{-9}	16.149
150 – 180	150	2.070×10^{-9}	22.523
180 – 200	180	5.464×10^{-10}	29.740
200 – 250	200	2.784×10^{-10}	37.105
250 – 300	250	7.248×10^{-11}	45.546
300 – 350	300	2.418×10^{-11}	53.628
350 – 400	350	9.518×10^{-12}	53.298
400 – 450	400	3.725×10^{-12}	58.515
450 – 500	450	1.585×10^{-12}	60.828
500 – 600	500	6.967×10^{-13}	63.822
600 – 700	600	1.454×10^{-13}	71.835
700 – 800	700	3.614×10^{-14}	88.667
800 – 900	800	1.170×10^{-14}	124.64
900 – 1000	900	5.245×10^{-15}	181.05
1000–	1000	3.019×10^{-15}	268.00

3. Solution and Calculation of Orbital Elements

We have simulated the differential equation (12) and hence obtained different orbital elements. Grewal, Weill and Andrews (2007) calculated the orbital elements as follows:

1. Specific angular momentum: $\vec{h} = \vec{r} \times \vec{v}$,
2. Specific orbital energy: $\epsilon = \frac{v^2}{2} - \frac{\mu}{2}$,
3. Eccentricity: $e = \sqrt{1 + \frac{2\epsilon h^2}{\mu^2}}$,
4. $l = \frac{h^2}{\mu}$,
5. Semi major axis: $a = \frac{l}{1-e^2}$,
6. Periodic time: $T = 2\pi\sqrt{\frac{a^3}{\mu}}$,
7. Angle of inclination: $i = \cos^{-1}\left(\frac{h_z}{h}\right)$,
8. Vector towards ascending node: $\vec{n} = \vec{k} \times \vec{h}$,
9. Longitude of ascending node: $\Omega = \cos^{-1}\left(\frac{n_x}{|n|}\right)$, if $n_y < 0$ then $\Omega = 2\pi - \Omega$,
10. Eccentricity vector: $\vec{e} = \frac{v^2\vec{r}}{\mu} - \frac{(\vec{r}\cdot\vec{v})\vec{v}}{\mu} - \frac{\vec{r}}{|\vec{r}|}$,
11. Argument of peripsis: $\omega = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{|\vec{n}|\cdot|\vec{e}|}\right)$, if $n_z < 0$ then $\omega = 2\pi - \omega$,
12. True anomaly: $f = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{|\vec{r}|\cdot|\vec{e}|}\right)$,
13. Eccentric anomaly: $E = \cos^{-1}\left(\frac{e+\cos f}{1+e\cos f}\right)$,
14. Mean anomaly: $M = E - e\sin E$.

We have fixed the initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ in kilometers, ballistic coefficient $B^* = 50 \text{ kg/m}^2$ and considered three different initial velocities (i) $\vec{v}_0 = [7.6, 0, 0]$,

(ii) $\vec{v}_0 = [7.7, 0, 0]$ and (iii) $\vec{v}_0 = [7.8, 0, 0]$ in km/sec^2 . For each of these three initial velocities which lead to low Earth orbit we have simulated the system of differential equations (12). We have analyzed each of three cases till satellite hits on the Earth. The minimum and maximum values of orbital elements for each of these three initial velocities over different time periods are shown in table-2, table-3 and table-4 respectively.

Table 2: $\vec{r}_0 = [0, -5888.9727, -3400]$ km ; $\vec{v}_0 = [7.6, 0, 0]$ km/sec ; $B^* = 50$ kg/m^2 ; Satellite collapses after 219.486111111 days.

Orbital Elements	1 Day		30 Days		180 Days		219 Days	
	Min	Max	Min	Max	Min	Max	Min	Max
a	6.7010×10^3	6.7069×10^3	6.6730×10^3	6.7069×10^3	6.5111×10^3	6.7069×10^3	6.4632×10^3	6.7069×10^3
e	0.0144	0.0161	0.0138	0.0164	0.0118	0.0164	0.0113	0.0164
i	0.5239	0.5239	0.5239	0.5239	0.5242	0.5242	0.5237	0.5237
Ω	0	6.2829	0	6.2829	0	6.2832	0	6.2832
ω	1.4933	1.8530	4.8377×10^{-4}	6.2830	5.5913×10^{-5}	6.2830	5.5913×10^{-5}	6.2830
f	6.7882×10^{-4}	3.1416	1.9413×10^{-4}	3.1416	4.3944×10^{-5}	3.1416	4.3944×10^{-5}	3.1416

Table 3: $\vec{r}_0 = [0, -5888.9727, -3400]$ km ; $\vec{v}_0 = [7.7, 0, 0]$ km/sec ; $B^* = 50$ kg/m^2 ; Satellite collapses after 453.954166667 days.

Orbital Elements	1 Day		30 Days		180 Days		453 Days	
	Min	Max	Min	Max	Min	Max	Min	Max
a	6.8781×10^3	6.8836×10^3	6.8555×10^3	6.8836×10^3	6.7284×10^3	6.8836×10^3	6.4315×10^3	6.8836×10^3
e	0.0101	0.0117	0.0095	0.0120	0.0085	0.0120	0.0062	0.0120
i	0.5242	0.5242	0.5242	0.5242	0.5236	0.5236	0.5239	0.5239
Ω	0	6.2829	0	6.2829	0	6.2832	0	6.2832
ω	4.6140	5.0028	6.1948×10^{-5}	6.2825	6.1948×10^{-5}	6.2832	1.2555×10^{-5}	6.2832
f	0	3.1359	0	3.1415	0	3.1416	0	3.1416

Table 4: $\vec{r}_0 = [0, -5888.9727, -3400]$ km ; $\vec{v}_0 = [7.8, 0, 0]$ km/sec ; $B^* = 50$ kg/m^2 ; Satellite collapses after 620.851388889 days.

Orbital Elements	1 Day		30 Days		180 Days		620 Days	
	Min	Max	Min	Max	Min	Max	Min	Max
a	7.0670×10^3	7.0725×10^3	7.0483×10^3	7.0725×10^3	6.9442×10^3	7.0725×10^3	6.5423×10^3	7.0725×10^3
e	0.0366	0.0381	0.0358	0.0384	0.0330	0.0384	0.0232	0.0384
i	0.5239	0.5239	0.5241	0.5241	0.5236	0.5236	0.5237	0.5237
Ω	0	6.2829	0	6.2829	0	6.2832	0	6.2832
ω	4.6913	4.9068	1.2484×10^{-4}	6.2830	9.8978×10^{-7}	6.2832	9.8978×10^{-7}	6.2832
f	0	3.1411	0	3.1414	0	3.1416	0	3.1416

The orbit of the satellite for satellite with initial position $\vec{r}_0 = [0, -5888.9727, -3400] \text{ km}$, initial velocity $\vec{v}_0 = [7.8, 0, 0] \text{ km/sec}$ and ballistic coefficient $B^* = 50 \text{ kg/m}^2$; for 1 day, 3 days and 7 days are shown in figure 1 respectively from left to right.

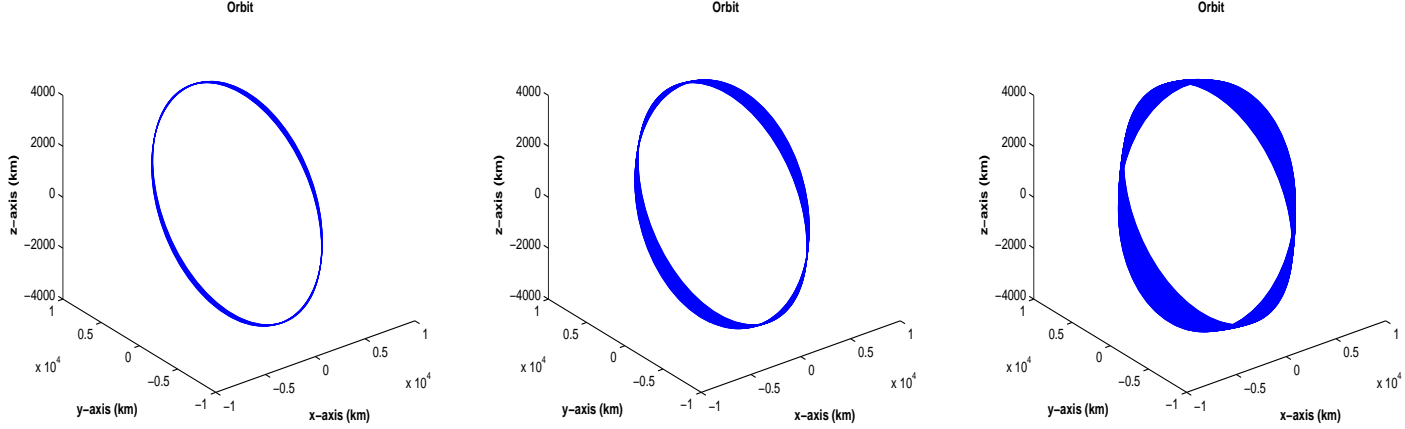


Fig. 1.— Orbit of Satellite 1 day, 3 days and 7 days from left to right

The graphs of angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semi major axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400] \text{ km}$, initial velocity $\vec{v}_0 = [7.8, 0, 0] \text{ km/sec}$ and ballistic coefficient $B^* = 50 \text{ kg/m}^2$; for 1 day, 3 days and 7 days are shown in figure 2, figure 3 and figure 4 respectively.

4. Discussion and Concluding Remarks

The equations governing motion of the satellite under the oblateness of Earth and atmospheric drag have been simulated. In table-2 to table-4 minimum and maximum values of orbital elements over a different time period have been reported. From these tables it can be seen that satellite will sustain for longer time in low Earth orbit if initial velocity is $\vec{v}_0 = [7.8, 0, 0] \text{ km/sec}$, initial position $\vec{r}_0 = [0, -5888.9727, -3400] \text{ km}$ and ballistic

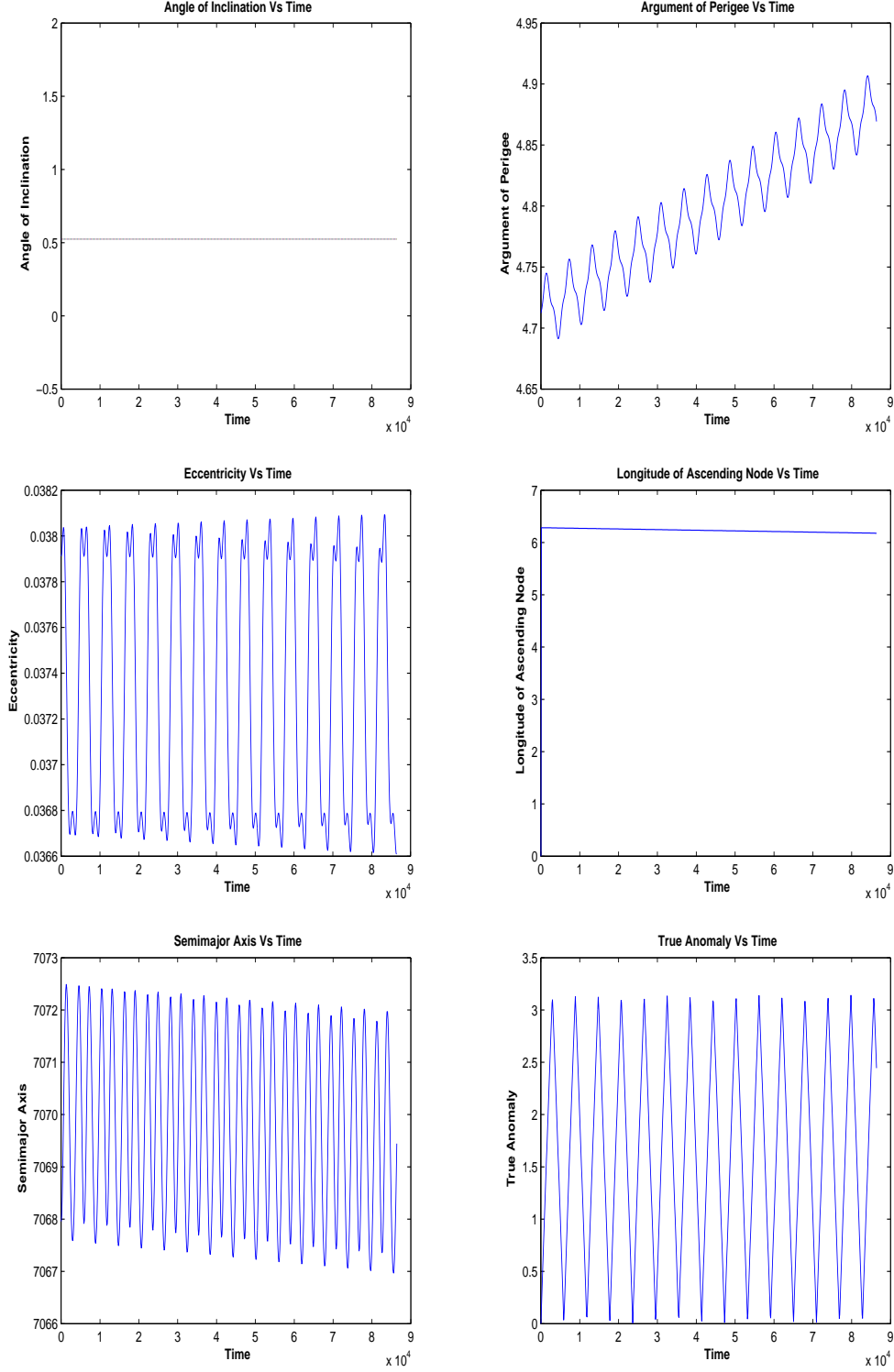


Fig. 2.— Angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semi major axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ km, initial velocity $\vec{v}_0 = [7.8, 0, 0]$ km/sec and $B^* = 50$ kg/m²; for 1 day

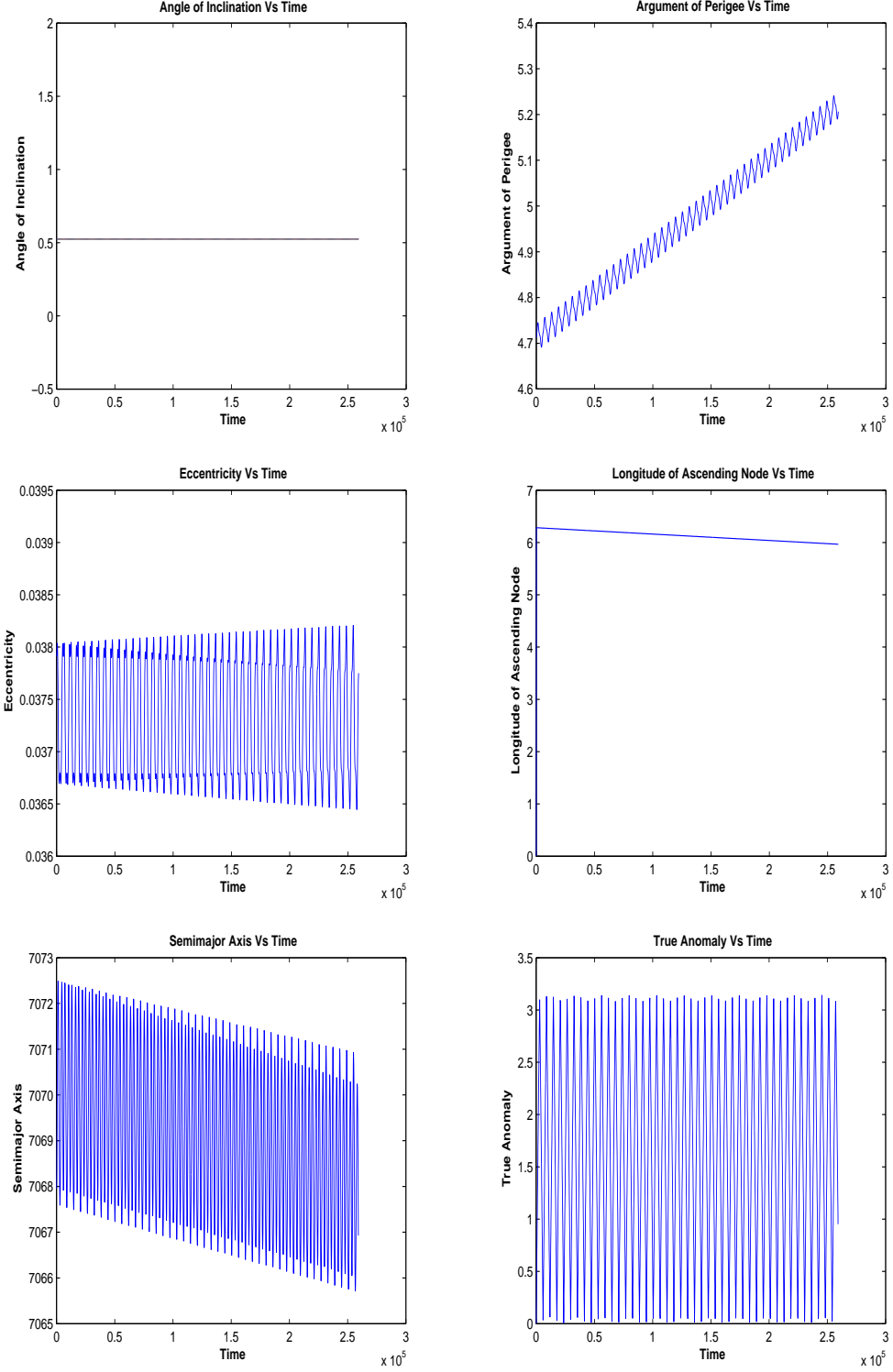


Fig. 3.— Angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semi major axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ km, initial velocity $\vec{v}_0 = [7.8, 0, 0]$ km/sec and ballistic coefficient $B^* = 50$ kg/m²; for 3 days

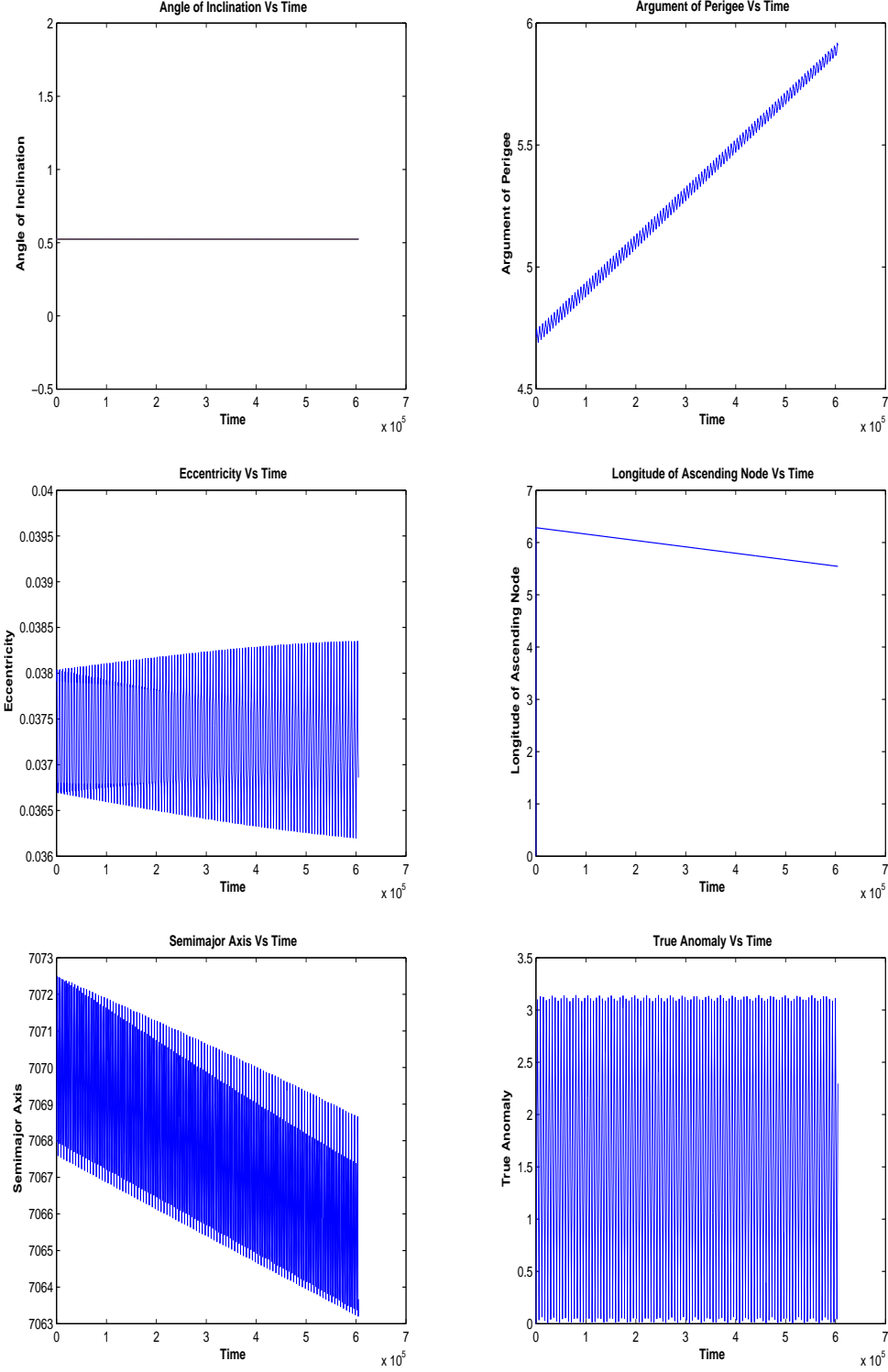


Fig. 4.— Angle of inclination, argument of perigee, eccentricity, longitude of ascending node, semi major axis and true anomaly with initial position $\vec{r}_0 = [0, -5888.9727, -3400]$ km, initial velocity $\vec{v}_0 = [7.8, 0, 0]$ km/sec and ballistic coefficient $B^* = 50$ kg/m²; for 7 days

coefficient $B^* = 50 \text{ kg/m}^2$; From figure 1, it can be seen that even for a shorter period of time (1 day, 3 days and 7 days), oblateness of earth and atmospheric drag effects the orbit of the satellite. From figure 2 to figure 4, we have observed following salient features of simulation:

1. The variation of argument of perigee and longitude of ascending node is almost linear.
2. The eccentricity increases and then decreases over a longer time.
3. The true anomaly varies between 0 to 3.1416.
4. There is significant decline in semi-major axis.
5. For initial position $\vec{r}_0 = [0, -5888.9727, -3400] \text{ km}$, initial velocity $\vec{v}_0 = [7.8, 0, 0] \text{ km/sec}$ and ballistic coefficient $B^* = 50 \text{ kg/m}^2$ satellite collapses on Earth after 620.85138889 days, since the height of satellite from surface of Earth on 620th day is approximately 6 km.

The analysis suggest that in order to have a satellite in low Earth orbit under the effect of oblateness of Earth and atmospheric drag, it is necessary to put a control in the motion of satellite.

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